

Optimization of a dual free piston Stirling engine

J. Boucher, F. Lanzetta *, P. Nika

Institut FEMTO-ST, CNRS UMR 6174, Département CREST Parc Technologique, 2 Avenue Jean Moulin, 90000 Belfort, France

Abstract

This work relates the theoretical study of the dynamic behavior of a dual free-piston Stirling engine (DFPSE) coupled with an asynchronous linear alternator. This machine integrates one piston and two displacers placed in a symmetrical way compared to the piston to improve the stability of the machine. The paper presents an analytical study of the dynamic balance equations of a DFPSE. This model takes into account the non-linear dissipative effects of the fluid and the electromagnetic forces. The dynamic balance equations of the machine are solved by means of linearized pressure in the time domain especially. The objective is to evaluate the thermo-mechanical conditions for stable operation of the engine. The developed model may be used to simulate the dynamic behaviour of a built engine. The DFPSE produces a mechanical power of 1 kW and it has a design operating point of 1.4 MPa corresponding to the frequency about 22 Hz. Helium is the working fluid. This machine is designed to be used as a micro combined heat and power (μ CHP) system for combined generation of electricity and heat.

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1. Introduction

Currently, there is a strong and media pressure for development systems of energy production and transformation which are powerful with a durable policy of development [1]. The growing worldwide demand for less polluting form of energy has led to a renewed interest in the use of cogeneration technologies [2]. The concept of micro combined heat and power (μ CHP) or micro cogeneration has been known for long time. Cogeneration systems have the ability to produce both useful thermal energy and electricity from a single source of fuel such as oil or natural gas.

There is an increasing interest in the use of Stirling engine based cogeneration systems for residential and commercial because of their prospect for high efficiency, good performance at partial load, fuel flexibility, low emission level, low vibration and noise level [3]. The Stirling engine is an external combustion reciprocating engine developed by

Robert Stirling in 1816. It uses an external source of energy to heat the gas located inside a cylinder (chamber). This gas, under pressure, expands when heated, driving a piston to perform work. The expanded gas volume, having released much of its energy, is cooled and compressed before the next heating cycle. Regenerator is the principal element of the Stirling engine. It is a porous environment which must absorb and reject heat during compression and expand gas volume. Stirling cycle is more near Carnot cycle when this element is effective [4,5]. Stirling engines are classified according to their arrangement [6]: the *Alpha*, *Beta* and *Gamma*. The *Alpha* configurations have two pistons in separate cylinders connected in series by a regenerator, a heater and a cooler. Both the *Beta* and the *Gamma* configurations use the displacement pistons arrangement, but the *Beta* arrangement has the piston and the displacer in the same cylinder while the *Gamma* arrangement uses different cylinders. For this study, we considered a Beta configuration (Fig. 1). This paper presents in the first part the mechanical characteristics of a dual free piston Stirling engine (DFPSE). Then the mathematical model is considered stating assumptions relative to the dynamical forces balance applied to the

* Corresponding author. Tel.: +33 3 8457 8224; fax: +33 3 8457 0032.
E-mail address: francois.lanzetta@univ-fcomte.fr (F. Lanzetta).

Nomenclature

A	cross-sectional area for moving element, m^2	δ	coefficient of real part of the polynomial equation
A'	internal cross-sectional area, m^2	χ	coefficient of imaginary part of the polynomial equation
C_{Hd}	displacer gas spring damping per mass unit, N s m^{-1}	ΔP	pressure drop, Pa
C_{Hp}	piston gas spring damping per mass unit, N s m^{-1}	ε	coefficient of imaginary part of the polynomial equation
C_{d}	displacer load damping per mass unit, N s m^{-1}	γ	isentropic coefficient
C_{p}	piston load damping per mass unit, N s m^{-1}	γ	coefficient of real part of the polynomial equation
d	diameter, mm	ψ	calculation parameter
D	damping coefficient per mass unit, s^{-1}	ϕ	piston-displacer phase angle, $^\circ$
$[D]$	matrix of damping coefficient	ω	angular velocity, rad s^{-1}
E	calculation parameter	ξ	coefficient of imaginary part of the polynomial equation
f	frequency, Hz		
f_{cut}	cut-off frequency, Hz		
F	external force per mass unit, m s^{-2}		
$[F]$	matrix of external forces		
j	imaginary number, $\sqrt{-1}$		
K	stiffness coefficient per mass unit, s^{-2}		
$[K]$	matrix of stiffness coefficient		
m	mass, kg		
P	pressure, bar		
Q	heat transfer, W		
$\dot{Q}(\omega)$	polynomial equation		
$\dot{Q}$	power, W		
r_1, r_2	displacer–piston stroke ratio, $Y_{\text{d}}/Y_{\text{p}}$		
r	working gas (helium) constant, $\text{J kg}^{-1} \text{K}^{-1}$		
S	calculation parameter		
t	time, s		
T	temperature, K		
V	volume, m^3		
y	displacement, m		
y_{de}	expansion clearance space, mm		
y_{pc}	compression clearance space, mm		
Greek symbols			
α	coefficient of real part of the polynomial equation		
β	coefficient of real part of the polynomial equation		
		Subscripts	
		alt	alternator
		b	bounce space
		c	compression space
		ch	charge
		cool	cooling
		d	displacer
		e	expansion space
		g	gas
		h	hot
		H	gas spring hysteresis losses
		Im	imaginary part
		i	initial condition
		k	cold
		mec	mechanical
		p	piston
		Re	real part
		reg	regenerator
		th	total heat power
		0	initial condition
		y_{de}	expansion clearance space, m
		y_{pc}	compression clearance space, m

mechanical elements and their thermofluidic equivalent elements. Optimized geometrical (volumes, masses) and thermophysical (pressures, temperatures) parameters are calculated to obtain a stable running of the machine designed to produce a mechanical power of 1 kW.

2. Dual free piston Stirling engine (DFPSE) analysis

2.1. Description of the DFPSE

Stirling engines are beginning to stage a comeback to the market since the development of the modern free piston Stirling engine [7]. Free piston Stirling engine was invented

by Beale in 1964 [8]. The free piston engine technology based on the *Beta* configuration was developed to alleviate the technical barrier posed by leakage problems. Free piston Stirling engines are expected to eliminate mechanical contact, friction and wear, and provide tight sealing of the casing, thus requiring no mechanical maintenance during an operating lifetime of about 10 years [8,9]. Free piston Stirling engine use working gas like gas spring in order to give an adequate movement to the various elements of the engine (piston and displacer).

A double free piston Stirling engine (DFPSE) arrangement is shown in Fig. 2. The engine acts as a vibrator generator in which gas is compressed and expanded by

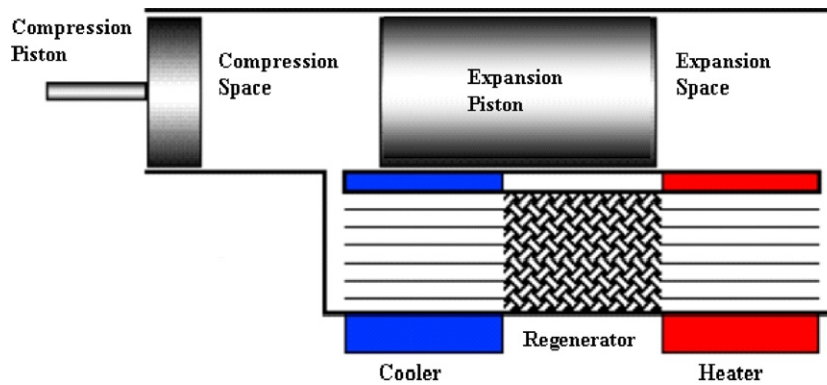


Fig. 1. Beta Stirling engine.

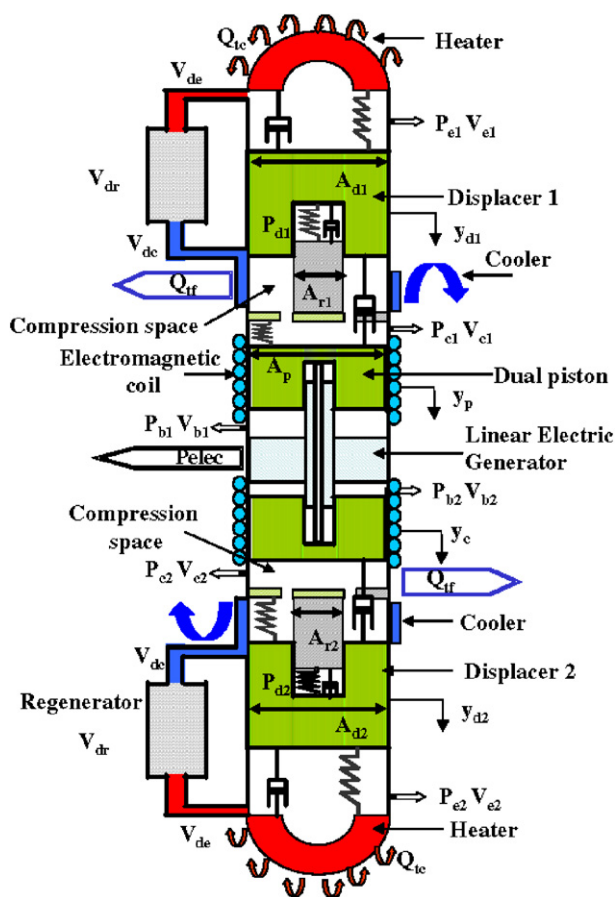


Fig. 2. DFPSE system.

oscillating system of masses and gas springs. The engine appears as a close gas filled vessel with three moving parts: a piston and two displacers. The engine presents two heads of which each one is heated by an external heat sources. The two gas spaces enclosed by the displacers and the heads of the vessel are the expansion spaces which temperature can reach 400–700 °C. The working gas (helium, hydrogen, air) acts as the lubricant. The engine has a compression piston with double effect (the dual piston) and two displacers laid out in a symmetrical way compared

to the dual one. The dual piston moving acts directly on each adjacent compression space. The two gas spaces enclosed by the displacers and the dual piston faces are water-cooled at temperature of 40–90 °C. The gradient pressures between hot and cold spaces strongly influence the dynamic behavior of the machine. The power strike is provided by the force enacted by the expanded gas in the two hot spaces on the displacers which are coupled to the dual power piston by gas springs only. Electricity is generated by the interaction between piston and a linear alternator. This interaction causes an electromagnetic force with viscous effect which depends on the piston velocity. The engine works with a rating of natural oscillations. In the DFPSE, there are no mechanical linkages coupling the pistons or displacers. The motions of those components follow working gas pressure variations. The free piston with the attached linear alternator can be tightly sealed to prevent the leakage of the working gas for a substantial period of time. The buffer volume (spring gas) present under the compression piston makes it possible to place all the electric device of electricity generation. Hysteresis losses of cycle are reduced and we can consider a system which functions in trigeneration (electricity, heat, cold).

The major advantages of free piston engines include input and output versatility, quiet operation, zero wear, zero maintenance, long life, ease of interfacing with the electric grid, continuous power operation and potential for high efficiency. The major disadvantage is the difficulty to stabilize the movements of the different elements like dual piston and displacers. The goal of this work is to find the mechanical and thermodynamical parameters which allow a correct operation of the DFPSE designed to produce a mechanical power of 1 kW.

2.2. Generalized analysis

From a mechanical mass-spring analogy (Figs. 2 and 3), we performed a dynamic study of a free piston Stirling engine. Various gas volume of the engine behave like springs and shock absorbers. Compressive forces action

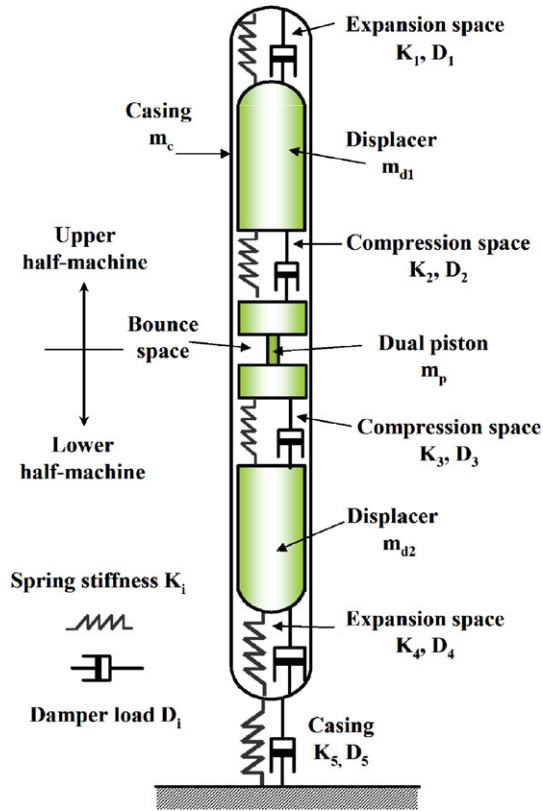


Fig. 3. Mechanical analogy.

on piston faces and displacer faces are integrated into the forces balance. Fig. 3 shows the dynamic behaviour of a free piston Stirling engine with connection spring stiffness K and damping coefficient D between piston mass m_p and displacer masses m_{d1} and m_{d2} . The external mass of the engine casing m_c is higher than the other moving masses (piston and displacers) and the effects will be neglected in the mechanical forces balance. Then, the DFPSE may be described by a viscous damped three degree of freedom spring-mass system under external forces [10,4]. The dynamic equations may also be written as the sum of the spring factor matrix $[K]$ and the damping factor matrix $[D]$ [10]. The effects of the gravitational field have also been neglected because they are quite small compared to the effects of the pressures involved in the machine [11].

The system of coupled differential equations describes the dynamic behavior of the DFPSE (Fig. 3). Model outputs available are evolution pressure, displacer and piston displacement, the electric charge, the thermal and electrical power. The motion of the system is completely described by the positions y_i of the respective piston and displacers at any time t from the respective equilibrium positions. The dynamic behavior of a DFPSE is presented in the matrix form (Eq. (1)), a matrix of the stiffness coefficients (Eq. (2)) and a matrix of the damping coefficients (Eq. (3)). These matrices are written per unit of moving element mass of the machine.

Newton's second law yields:

$$\begin{bmatrix} \ddot{y}_p \\ \ddot{y}_{d1} \\ \ddot{y}_{d2} \end{bmatrix} = [K] \begin{bmatrix} y_p \\ y_{d1} \\ y_{d2} \end{bmatrix} + [D] \begin{bmatrix} \dot{y}_p \\ \dot{y}_{d1} \\ \dot{y}_{d2} \end{bmatrix} + [F(t)] \quad (1)$$

$$[K] = \begin{bmatrix} K_{pp} & K_{pd1} & K_{pd2} \\ K_{d1p} & K_{d1d1} & K_{d1d2} \\ K_{d2p} & K_{d2d1} & K_{d2d2} \end{bmatrix} \quad (2)$$

$$[D] = \begin{bmatrix} D_{pp} & D_{pd1} & D_{pd2} \\ D_{d1p} & D_{d1d1} & D_{d1d2} \\ D_{d2p} & D_{d2d1} & D_{d2d2} \end{bmatrix} \quad (3)$$

$$[F(t)] = \begin{bmatrix} F_p(t) & 0 & 0 \\ 0 & F_{d1}(t) & 0 \\ 0 & 0 & F_{d2}(t) \end{bmatrix} \quad (4)$$

$[F(t)]$ represents the matrix of possible external forces (i.e. piloting). In this study neither the piston, nor the displacers are controlled and $F_p(t) = 0$, $F_{d1}(t) = 0$ and $F_{d2}(t) = 0$. The piston is submitted to an electromagnetic force corresponding to the linear alternator function only. The pressure gradients existing between the different moving elements (piston and displacers) strongly influence the dynamic behavior of the machine. Each applied force has a component normal to the displacement surface pointing up. The variation of the volume located in the hot part (expansion space) is related to the displacement of the displacer while that located in the cold part (compression space) will be related to the displacement of the piston.

The differential model represents the damped dynamic system with three degrees of freedom (Figs. 2 and 3) [8,12].

$$\begin{bmatrix} \ddot{y}_p \\ \ddot{y}_{d1} \\ \ddot{y}_{d2} \end{bmatrix} = [K] \begin{bmatrix} y_p \\ y_{d1} \\ y_{d2} \end{bmatrix} + [D] \begin{bmatrix} \dot{y}_p \\ \dot{y}_{d1} \\ \dot{y}_{d2} \end{bmatrix} \quad (1)$$

The equations of motion of the different elements are for the piston:

$$m_p \ddot{y}_p = A_p(P_{c1} - P_{c2}) - A'_p(P_{b1} - P_{b2}) - (C_{palt} + C_{Hp})\dot{y}_p \quad (5)$$

the displacer 1:

$$m_{d1} \ddot{y}_{d1} = A_d(P_{c1} - P_{d1}) - A_r(P_{c1} - P_{d1}) - (C_{d1alt} + C_{Hd1})\dot{y}_{d1} \quad (6)$$

the displacer 2:

$$m_{d2} \ddot{y}_{d2} = A_d(P_{c2} - P_{e2}) - A_r(P_{d2} - P_{c2}) - (C_{d2alt} + C_{Hd2})\dot{y}_{d2} \quad (7)$$

The dynamic equations take into account of the interaction between the alternator and the pistons (the electric charge causes a braking force).

The DFPSE is divided into two parts (Fig. 3). The upper half-machine contains the expansion space (K_1, D_1), the displacer (m_{d1}) and the compression space (K_2, D_2). The lower

half-machine integrates the compression space (K_3, D_3), the displacer (m_{d2}) and the expansion space (K_4, D_4). The dual piston m_p separates these two parts. In order to approximate fairly good the reality, the isothermal Schmidt analysis is applied in order to linearize the pressure in the different working volumes of the machine (Figs. 2 and 3). If the total gas mass contained in the machine is known, the principle of conservation makes it possible within the framework of the assumption of isotherm of gas volumes to determine the pressures P and P_i . The pressure P corresponds to the mean pressure in the gas and P_i the initial pressure just before oscillation of the piston and displacers start ($y_d = y_p = 0$). These pressures appear in their general formulations:

$$P = \frac{mr}{S} [1 + \psi]^{-1} \quad (8)$$

$$P_i = P_{ch} \left(\frac{S_0}{S} \right) \quad (9)$$

$$S_0 = \frac{A_p y_{pc}}{T_k} + \frac{V_k}{T_k} + \frac{V_r}{T_k} + \frac{V_h}{T_k} + \frac{A_d y_{de}}{T_k} \quad (10)$$

The mass of gas m , the initial pressure P_i before oscillation start and the two parameters ψ and S are written for each part of the machine. P_{ch} is the pressure of the gas charged in the engine at ambient temperature before oscillation start. This pressure integrates the S_0 parameter which corresponds to the initial condition before start when the temperature T_k is the same in the entire volume of the machine.

The mean gas temperature T_r in the two regenerator of the machine (upper and lower half-machine) will be evaluated by the average logarithm of the temperatures of their corresponding hot (T_h) and cold (T_k) ends:

$$T_r = \frac{T_h - T_k}{\ln(T_h/T_k)} \quad (11)$$

2.3. Linearization of the pressure in the machine

We consider the upper and lower half-machine volumes (Fig. 3). The expressions of the pressures in these different volumes are linearized in order to be used in the dynamic analysis (Eq. (8)). Referring to Fig. 2, the different volumes variations are as follows:

- For the volume of the upper half-machine:

$$V_{b1} = (A'_{p1} - A'_{r1}) y_{p1} \quad (12)$$

$$V_{c1} = A_{p1}(y_{p1} + y_{pc1}) - (A_{d1} - A_{r1}) y_{d1} \quad (13)$$

$$V_{e1} = A_{d1}(y_{d1} + y_{de1}) \quad (14)$$

- For the volume of the lower half-machine:

$$V_{b2} = (A'_{p2} + A'_{r2}) y_{p2} \quad (15)$$

$$V_{c2} = A_{p2}(y_{pc2} - y_{p2}) + (A_{d2} - A_{r2}) y_{d2} \quad (16)$$

$$V_{e2} = A_{d2}(y_{de2} - y_{d2}) \quad (17)$$

(y_{de1} and y_{de2}) and (y_{pc1} and y_{pc2}) are the heights of the expansion and compression volumes 1 and 2 at initial condition before oscillations, respectively. We can consider the following conditions of symmetry concerning the geometry of the DFPSE: $A_{p1} = A_{p2} = A_p$ and the displacements of the pistons: $y_{p1} = y_{p2} = y_p$.

For the volume of the upper half-machine, the mean pressure is

$$P_1 = \frac{m_1 r}{S_1} [1 + \psi_1]^{-1} \quad (18)$$

$$P_{1i} = P_{ch} \left(\frac{S_0}{S_1} \right) \quad (19)$$

with

$$S_1 = \frac{A_p y_{pc1}}{T_{k1}} + \frac{V_{k1}}{T_{k1}} + \frac{V_{r1}}{T_{r1}} + \frac{V_{h1}}{T_{h1}} + \frac{A_{d1} y_{de1}}{T_{h1}} \quad (20)$$

$$\psi_1 = \frac{A_p y_p}{T_{k1} S_1} + \left(\frac{A_{r1} - A_{d1}}{T_{k1} S_1} + \frac{A_{d1}}{T_{h1} S_1} \right) y_{d1} \quad (21)$$

The mean temperature of the regenerator of the upper half-machine is

$$T_{r1} = \frac{T_{h1} - T_{k1}}{\ln(T_{h1}/T_{k1})} \quad (22)$$

Then, after a binomial expansion of $[1 + \psi_1]^{-1}$, neglecting the second-order terms, the linearized pressure becomes:

$$P_1 = P_{1i} \left[1 - \frac{A_p y_p}{T_{k1} S_1} + \left(\frac{A_{d1} - A_{r1}}{T_{k1} S_1} - \frac{A_{d1}}{T_{h1} S_1} \right) y_{d1} \right] \quad (23)$$

$$P_{b1} = P_{1i} \left(1 + \frac{\gamma A'_{p1}}{V_{b1i}} y_p \right) \quad (24)$$

$$P_{c1} \approx P_1 \quad (25)$$

$$P_{d1} = P_{1i} \left(1 + \frac{\gamma A_{r1}}{V_{d1i}} y_{d1} \right) \quad (26)$$

$$P_{e1} \approx P_1 + \Delta P_1 = P_1 + \frac{C_{p1}}{A_{d1}} \dot{y}_p + \frac{C_{d1}}{A_{d1}} \dot{y}_{d1} + \frac{E_{p1}}{A_{d1}} y_p + \frac{E_{d1}}{A_{d1}} y_{d1} \quad (27)$$

For the lower half-machine, the mean pressure is

$$P_2 = \frac{m_2 r}{S_2} [1 + \psi_2]^{-1} \quad (28)$$

$$P_{2i} = P_{ch} \left(\frac{S_0}{S_2} \right) \quad (29)$$

with

$$S_2 = \frac{A_p y_{pc2}}{T_{k2}} + \frac{V_{k2}}{T_{k2}} + \frac{V_{r2}}{T_{r2}} + \frac{V_{h2}}{T_{h2}} + \frac{A_{d2} y_{de2}}{T_{h2}} \quad (30)$$

$$\psi_2 = \frac{A_p y_p}{T_{k2} S_2} + \left(\frac{A_{r2} - A_{d2}}{T_{k2} S_2} + \frac{A_{d2}}{T_{h2} S_2} \right) y_{d2} \quad (31)$$

The mean temperature of the regenerator of the lower half-machine is

$$T_{r2} = \frac{T_{h2} - T_{k2}}{\ln(T_{h2}/T_{k2})} \quad (32)$$

Then, after a binomial expansion of $[1 + \psi_2]^{-1}$, neglecting the second-order terms, the linearized pressure becomes:

$$P_2 = P_{2i} \left[1 - \frac{A_p y_p}{T_{k2} S_2} + \left(\frac{A_{d2} - A_{r2}}{T_{k2} S_2} - \frac{A_{d2}}{T_{h2} S_2} \right) y_{d2} \right] \quad (33)$$

$$P_{b2} = P_{2i} \left(1 - \frac{\gamma A'_{p2}}{V_{b2i}} y_p \right) \quad (34)$$

$$P_{c2} \approx P_2 \quad (35)$$

$$P_{d2} = P_{2i} \left(1 - \frac{\gamma A_{r2}}{V_{d2i}} y_{d2} \right) \quad (36)$$

$$P_{e2} \approx P_2 + \Delta P_2 = P_2 - \frac{C_{p2}}{A_{d2}} \dot{y}_p - \frac{C_{d2}}{A_{d2}} \dot{y}_{d2} - \frac{E_{p2}}{A_{d2}} y_p - \frac{E_{d2}}{A_{d2}} y_{d2} \quad (37)$$

2.4. Stiffness $[K]$ matrix and damping $[D]$ matrix

The dynamic behavior of a FPSE is presented in the matrix form with a matrix of the stiffness coefficients as well as a matrix of the coefficient of dissipations. We consider the dynamic equations for a thermomechanical system with three degrees of freedom. Eqs. (5)–(7) may be presented in the matrix form:

$$\ddot{y}_p = K_{pp} y_p + K_{pd1} y_{d1} + K_{pd2} y_{d2} - D_{pp} \dot{y}_p \quad (38)$$

$$\ddot{y}_{d1} = K_{d1p} y_p + K_{d1d1} y_{d1} + D_{d1p} \dot{y}_p - D_{d1d1} \dot{y}_{d1} \quad (39)$$

$$\ddot{y}_{d2} = K_{d2p} y_p + K_{d2d2} y_{d2} + D_{d2p} \dot{y}_p - D_{d2d2} \dot{y}_{d2} \quad (40)$$

with

$$[K] = \begin{bmatrix} K_{pp} & K_{pd1} & K_{pd2} \\ K_{d1p} & K_{d1d1} & 0 \\ K_{d2p} & 0 & K_{d2d2} \end{bmatrix} \quad (41)$$

$$[D] = \begin{bmatrix} D_{pp} & 0 & 0 \\ D_{d1p} & D_{d1d1} & 0 \\ D_{d2p} & 0 & D_{d2d2} \end{bmatrix} \quad (42)$$

Different assumptions relate to the pressure within volumes and the initial pressure (Fig. 2).

The initial mean pressures within the upper and lower volumes are equal:

$$P_{li} = P_{2i} = P_i \quad (43)$$

The surface areas of the piston and the two displacers are equal:

$$A_{d1} = A_{d2} = A_d \quad (44)$$

The internal surface areas of the two displacers are equal:

$$A_{r1} = A_{r2} = A_r \quad (45)$$

The displacements of the piston and displacers are equal:

$$y_{pc1} = y_{pc2} = y_{pc} \quad (46)$$

$$y_{de1} = y_{de2} = y_{de} \quad (47)$$

The linearized coefficients of the stiffness and damping matrices are given by the Table 1. The damping load

matrix $[D]$ is composed by elements which generate equivalent viscous losses. C_{palt} corresponds to a loss of energy caused by a viscous dissipation of electromagnetic origin in the electromagnetic coil. C_{Hp} , C_{Hd1} and C_{Hd2} are hysteretic loss due to cyclic flows between the different elements (piston and displacers). These losses depend on the mass flow generated by the piston movements, the frequency and the magnitude of the pressure [13–17]. In this work the two displacers 1 and 2 are not controlled. Then, the corresponding damping coefficients do not exist and $C_{d1alt} = 0$ and $C_{d2alt} = 0$.

2.5. Analysis of the harmonic displacements of the pistons and displacers

We shall consider the piston and displacers 1 and 2 to be harmonic. Complex notation is used to describe the response of the components:

$$y_p = y_p e^{j(\omega t)}, \quad y_{d1} = y_{d1} e^{j(\omega t + \phi_1)}, \quad y_{d2} = y_{d2} e^{j(\omega t + \phi_2)} \quad (48a)$$

The time derivatives are:

$$\dot{y}_p = j\omega y_p e^{j(\omega t)}, \quad \dot{y}_{d1} = j\omega y_{d1} e^{j(\omega t + \phi_1)}, \quad \dot{y}_{d2} = j\omega y_{d2} e^{j(\omega t + \phi_2)} \quad (48b)$$

$$\ddot{y}_p = -\omega^2 y_p e^{j(\omega t)}, \quad \ddot{y}_{d1} = -\omega^2 y_{d1} e^{j(\omega t + \phi_1)}, \quad \ddot{y}_{d2} = -\omega^2 y_{d2} e^{j(\omega t + \phi_2)} \quad (48c)$$

Substituting Eqs. (48a)–(48c) into Eqs. (38)–(40) yields:

$$K_{pp} + r_1 K_{pd1} e^{j\phi_1} + r_2 K_{pd2} e^{j\phi_2} + j\omega D_{pp} + \omega^2 = 0 \quad (49)$$

$$K_{d1p} + r_1 K_{d1d1} e^{j\phi_1} + j\omega D_{d1p} + j\omega r_1 D_{d1d1} e^{j\phi_1} + \omega^2 r_1 e^{j\phi_1} = 0 \quad (50)$$

$$K_{d2p} + r_2 K_{d2d2} e^{j\phi_2} + j\omega D_{d2p} + j\omega r_2 D_{d2d2} e^{j\phi_2} + \omega^2 r_2 e^{j\phi_2} = 0 \quad (51)$$

We define:

- the ratio of the maximum displacement r_1 between the piston and the displacer 1:

$$r_1 = \frac{y_{d1max}}{y_{pmax}} \quad (52a)$$

$$r_1 = \frac{\sqrt{(\omega^2(K_{d1p} + D_{d1p}D_{d1d1}) + K_{d1d1}K_{d1p})^2 + \omega^2(\omega^2 D_{d1p} + K_{d1d1}D_{d1p} - K_{d1p}D_{d1d1})^2}}{(\omega^2 + K_{d1d1})^2 + (\omega^2 D_{d1d1})^2} \quad (52b)$$

- the ratio of the maximum displacements r_2 between the piston and the displacer 2:

$$r_2 = \frac{y_{d2max}}{y_{pmax}} \quad (53a)$$

$$r_2 = \frac{\sqrt{(\omega^2(K_{d2p} + D_{d2p}D_{d2d2}) + K_{d2d2}K_{d2p})^2 + \omega^2(\omega^2 D_{d2p} + K_{d2d2}D_{d2p} - K_{d2p}D_{d2d2})^2}}{(\omega^2 + K_{d2d2})^2 + (\omega^2 D_{d2d2})^2} \quad (53b)$$

Table 1
Elements of the stiffness and damping matrices

Elements of the stiffness matrix	Elements of the damping matrix
$K_{pp} = -\left(\frac{1}{S_2 T_k} + \frac{1}{S_1 T_k}\right) \frac{A_p^2}{m_p} P_i - \frac{2\gamma A_p^2}{m_p V_{b'}} P_i$	$D_{pp} = -\frac{1}{m_p} (C_{palt} + C_{Hp})$
$K_{pd1} = \frac{A_p}{m_p} P_i \left(\frac{A_{d1} - A_{r1}}{S_1 T_k} - \frac{A_{d1}}{S_1 T_{h1}} \right)$	$D_{pd1} = 0$
$K_{pd2} = \frac{A_p}{m_p} P_i \left(\frac{A_{d2} - A_{r2}}{S_2 T_k} - \frac{A_{d2}}{S_2 T_{h2}} \right)$	$D_{pd2} = 0$
$K_{d1p} = -\frac{A_p A_{r1}}{m_{d1}} P_i \frac{1}{T_k S_1} + \frac{E_{p1}}{m_{d1}}$	$D_{d1p} = \frac{C_{p1}}{m_{d1}}$
$K_{d2p} = -\frac{A_p A_{r2}}{m_{d2}} P_i \frac{1}{T_k S_2} + \frac{E_{p2}}{m_{d2}}$	$D_{d2p} = \frac{C_{p2}}{m_{d2}}$
$K_{d1d1} = \frac{A_{r1}}{m_{d1}} P_i \left(\frac{A_{d1} - A_{r1}}{T_k S_1} - \frac{A_{d1}}{T_{h1} S_1} - \frac{\gamma A_{r1}}{V_{d01}} \right) + \frac{E_{d1}}{m_{d1}}$	$D_{d1d1} = \frac{1}{m_{d1}} (C_{d1} - C_{Hd1} - C_{d1alt})$
$K_{d2d2} = \frac{A_{r2}}{m_{d2}} P_i \left(\frac{A_{d2} - A_{r2}}{T_k S_2} - \frac{A_{d2}}{T_{h2} S_2} - \frac{\gamma A_{r2}}{V_{d02}} \right) + \frac{E_{d2}}{m_{d2}}$	$D_{d2d2} = \frac{1}{m_{d2}} (C_{d2} - C_{Hd2} - C_{d2alt})$

- the phase ϕ_1 between the piston and the displacer 1:

$$r_1 e^{j\phi_1} = -\frac{K_{d1p} + j\omega D_{d1p}}{\omega^2 + K_{d1d1} + j\omega D_{d1d1}} \quad (54a)$$

$$\phi_1 = \tan^{-1} \left[-\frac{\omega(\omega^2 D_{d1p} + K_{d1d1} D_{d1p} - K_{d1p} D_{d1d1})}{\omega^2(K_{d1p} + D_{d1p} D_{d1d1}) + K_{d1d1} K_{d1p}} \right] \quad (54b)$$

- the phase ϕ_2 between the piston and the displacer 2:

$$r_2 e^{j\phi_2} = -\frac{K_{d2p} + j\omega D_{d2p}}{\omega^2 + K_{d2d2} + j\omega D_{d2d2}} \quad (55a)$$

$$\phi_2 = \tan^{-1} \left[-\frac{\omega(\omega^2 D_{d2p} + K_{d2d2} D_{d2p} - K_{d2p} D_{d2d2})}{\omega^2(K_{d2p} + D_{d2p} D_{d2d2}) + K_{d2d2} K_{d2p}} \right] \quad (55b)$$

The solution of the above system of differential Eqs. (49)–(51) is based on the roots of the polynomial equation $Q(\omega)$:

$$\begin{aligned} Q(\omega) = & -(\omega^2 + K_{pp} + j\omega D_{pp})(\omega^2 + K_{d1d1} + j\omega D_{d1d1}) \\ & \times (\omega^2 + K_{d2d2} + j\omega D_{d2d2}) + K_{pd1}(K_{d1p} + j\omega D_{d1p}) \\ & \times (\omega^2 + K_{d2d2} + j\omega D_{d2d2}) + K_{pd2}(K_{d2p} + j\omega D_{d2p}) \\ & \times (\omega^2 + K_{d1d1} + j\omega D_{d1d1}) = 0 \end{aligned} \quad (56)$$

This characteristics equation determines the 6 independent roots for the damped vibration problem and it can be put in the general complex form $Q(\omega)$ defined by the real part $Q_{Re}(\omega)$ and the imaginary part $Q_{Im}(\omega)$:

$$Q(\omega) = Q_{Re}(\omega) + jQ_{Im}(\omega) \quad (57)$$

$$Q_{Re}(\omega) = (\omega^6 - \beta\omega^4 + \delta\omega^2 - \alpha) \quad (58)$$

$$Q_{Im}(\omega) = (\chi\omega^5 - \varepsilon\omega^3 - \zeta\omega) \quad (59)$$

Sixth power of pulsation shows many oscillations modes are possible [10]. We study the auto excited periodic modes. For the free oscillations characterized by no external forces applied on the piston and on the two displacers ($F_p(t) = 0$, $F_{d1}(t) = 0$ and $F_{d2}(t) = 0$), the real part of the characteristic polynomial $Q_{Re}(\omega)$ is always of an order of power higher (6th degree) than the imaginary part of the characteristic polynomial $Q_{Im}(\omega)$ (5th degree). Steady oscillations appear when the characteristic polynomial has two imaginary roots and four roots with negative real parts [10,12]. Then, for a symmetrical machine in steady oscillations $Q_{Im}(\omega) = 0$ and the characteristic polynomial $Q(\omega)$ satisfy the following constraints [12–14]:

$$\alpha = D_{d1d1} + D_{pp} < 0 \quad (56a)$$

$$\beta = -K_{pp} - K_{d1d1} + D_{d1d1} D_{pp} > 0 \quad (56b)$$

$$\gamma = -D_{d1d1} K_{pp} - D_{pp} K_{d1d1} + 2D_{d1p} K_{pd1} < 0 \quad (56c)$$

$$\delta = K_{pp} K_{d1d1} - 2K_{d1p} K_{pd1} > 0 \quad (56d)$$

$$\beta - \frac{\gamma}{\alpha} - \frac{\delta\alpha}{\gamma} = 0 \quad (56e)$$

3. Results

A theoretical parametric study of a Stirling engine with a dual piston and two displacers is conducted to obtain the

frequencies and the operating capacities. The mechanical power can reach 1 kW for a frequency of 20–35 Hz. During dimensioning phase, internal dynamics of engine is actually unknown since no system mechanical imposes neither displacement of pistons, no phase lag between piston and displacer (on the contrary traditional Stirling engines whose rotational movement is recovered on the linkage) [18].

The matrix coefficients depend on the geometry of the engine. A linearized dynamical model, with stiffnesses matrix $[K]$ are fixed by the engine geometry and the average pressure gas. The three parameters of the damping load matrix D_{pp} , D_{d1d1} and D_{d2d2} depend on the constant electric charges and thermal dissipation coefficients in gas volume. E_{d1} , E_{d2} and E_p are corrective terms which multiply the pressure losses depending on the velocity and the displacement. The DFPSE characteristics are given in the Table 2.

Figs. 4–10 show the behavior of the FDSPE at stable operation with the different characteristics defined in Table 2 (Figs. 4 and 5). The charge pressure and the hot source temperate determine the mechanical power produced, the frequency of operation as well as the stroke of the displacer. The amplitude of the stroke and the phase between displacers and piston vary with the pressure. The strokes of the piston and displacers are 2 cm and 1 cm, respectively. The mean pressure in the machine is 1.4 MPa for a dynamic amplitude of the pressure of 0.1 MPa.

The curves of the Figs. 6–10 present the evolution of the mechanical power produced by the piston, the thermal power, the stroke of the piston, the amplitude ratio of behavior displacers and piston and finally the phase angle between the pistons and the displacers. The stable operations appear between 20 Hz and 35 Hz. For frequencies lower than 20 Hz, the operation of the engine is unstable and the amplitudes of piston and displacers grow strongly with unrealistic values. For frequencies higher than 35 Hz,

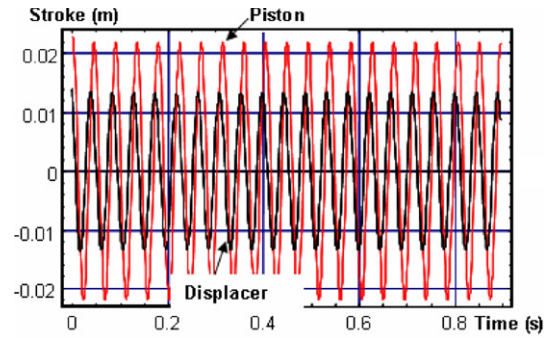


Fig. 4. Piston stroke for a frequency of 33 Hz.

it may be seen that it is impossible to extract power from the engine. For these frequencies the power losses by friction become very important and the different amplitudes of piston and displacers are damped until the mechanical power tends to zero. Fig. 6 shows the mechanical power of the engine versus frequency. With the increase of frequency, the mechanical power firstly increases to the maximum and then decreases gradually until zero power for the frequency of $f_{cut} = 37$ Hz which correspond to a cut-off mechanical power frequency. The maximum mechanical power is $\dot{Q}_{mec} = 1050$ W and the corresponding frequency is $f = 22.5$ Hz. Fig. 7 presents the thermal power of the machine versus frequency. This power corresponds to the input thermal power by an external source at one side of the machine only. The total input heat power of the FDSPE is the double. The maximum power decreases with the frequency and presents a maximum value $\dot{Q}_h = 1750$ W for the frequency $f = 20$ Hz. The total heat power is $\dot{Q}_{th} = 2\dot{Q}_h = 3500$ W. The cut-off frequency occurs for $f_{cut} = 37$ Hz. Fig. 8 shows the behaviour of the cooling power versus frequency. This power decreases with the frequency in the same order than the mechanical and thermal powers. The maximal cooling power is $\dot{Q}_{cool} = 2500$ W for the frequency $f = 20$ Hz. The cut-off frequency occurs for $f_{cut} = 37$ Hz. Figs. 6–8 show the conservation of the power in the system and:

$$\dot{Q}_{th} = 2\dot{Q}_h = \dot{Q}_{cool} + \dot{Q}_{mec} \quad (57)$$

Fig. 9 presents the displacement ratio versus frequency which is bell-shaped in the range of stable operation 20–37 Hz. The ratio of maximum displacement is $r_1 = y_{d2max}/y_{pmax} = 0.61$ for the frequency $f = 24.5$ Hz. This ratio of displacement does not appear for the same frequency than for the mechanical and thermal powers. This ratio of displacement varies from 0.52 to 0.61. The amplitude of displacement of the displacer is increasingly smaller than that of the piston. Fig. 10 indicates the relation between the phase angle between piston and displacers and the frequency. The movements of the displacers are supposed to be identical. The machine is built in a symmetrical way and we suppose $\varphi_1 = \varphi_2 = \varphi$. The phase angle decreases with the frequency and the optimum value $\varphi = -60^\circ$ (or

Table 2
DFPSE characteristics for the simulation

Dual free Stirling piston engine data	
General	
Working fluid	Helium
Gas constant	$r = 2080 \text{ J kg}^{-1} \text{ K}^{-1}$
Specific heat	$c_p = 5183 \text{ J kg}^{-1} \text{ K}^{-1}$
Isentropic coefficient	$\gamma = 1.667$
Dynamic viscosity	$\mu = 1.8 \times 10^{-5} \text{ Pa s}$
Charged pressure	$P_{ch} = 10 \text{ bar}$
Mean cold space temperature	$T_k = 320 \text{ K}$
Mean hot space temperature	$T_h = 815 \text{ K}$
Geometric	
Piston diameter	$d_p = 120 \text{ mm}$
Internal piston diameter	$d_{pi} = 116 \text{ mm}$
Diameter of the displacer rod	$d_r = 35 \text{ mm}$
Displacer diameter	$d_d = d_p$
Piston mass	$m_p = 6.2 \text{ kg}$
Displacer mass	$m_d = 0.426 \text{ kg}$
Height of the compression space at rest	$y_{pc} = 150 \text{ mm}$
Height of the expansion space at rest	$y_{de} = 100 \text{ mm}$

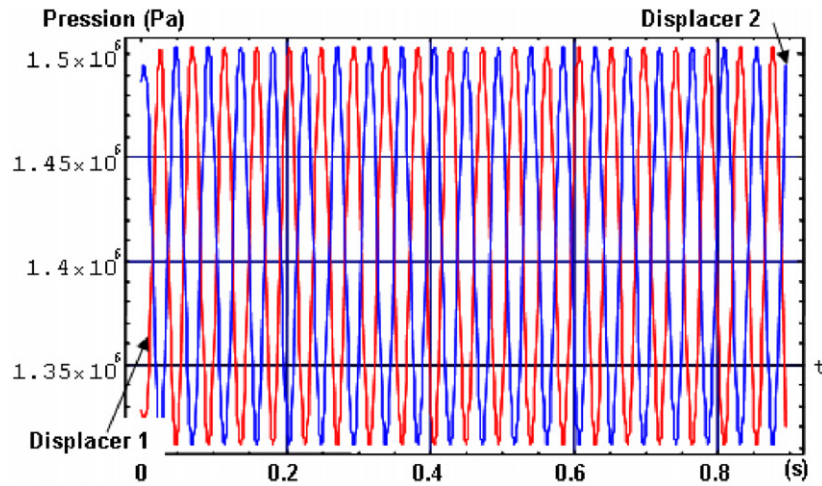


Fig. 5. Pressure amplitude for a frequency of 33 Hz.

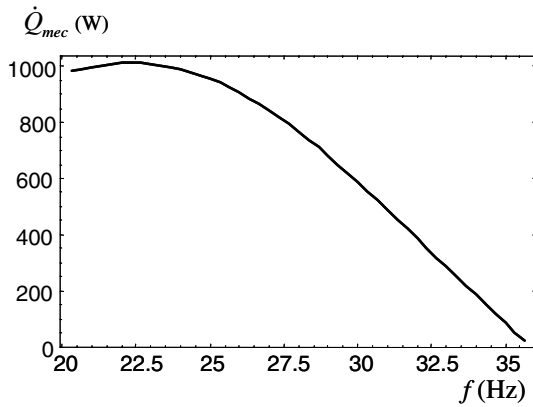


Fig. 6. Mechanical power versus frequency.

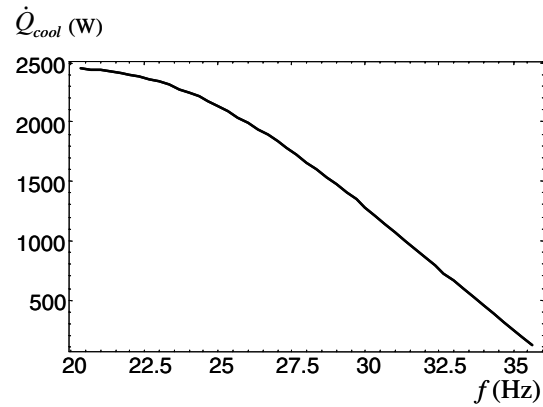


Fig. 8. Thermal power evacuated by the cold source versus frequency.

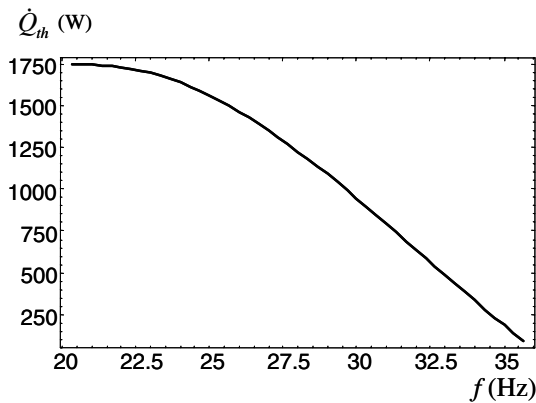
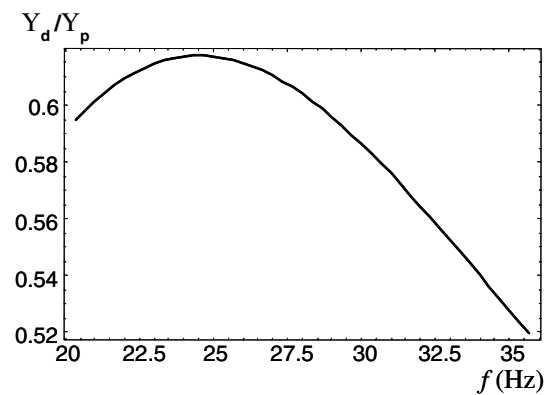


Fig. 7. Thermal power for one hot source only versus frequency.

Fig. 9. Amplitude ratio (y_d/y_p) versus frequency ($y_{d1} = y_{d2} = y_d$).

$\varphi = 300^\circ$) is obtained for 21 Hz. The negative value of the phase angle indicates piston motion lagging behind those of the displacers. For the stable operation of the machine, the piston-displacers phase angles lie in the fourth quad-

rant ($180^\circ < \varphi_{1,2} < 360^\circ$) depending of the size of the denominators in Eqs. (55a) and (55b). For a phase angle of 0° , it corresponds to a cut-off frequency of 35 Hz for which the DFPSE do not produce work.

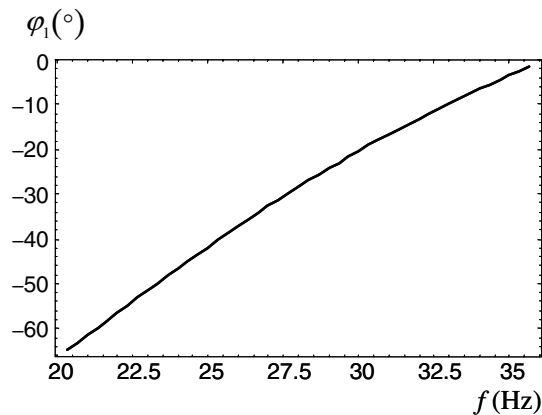


Fig. 10. Phase φ between piston and displacers versus frequency ($\varphi_1 = \varphi_2 = \varphi$).

4. Conclusions

The dynamic behaviour of a dual free-piston Stirling engine (DFPSE) coupled with an asynchronous linear alternator has been performed with a mathematical analysis including complex damping and stiffness coefficients. The dynamic balance equations of the machine are solved by means of linearized pressure in the time domain especially. The objective is to obtain a stable operation. This machine integrates one piston and two displacers placed in a symmetrical way compared to the piston to improve the mechanical stability of the machine. We studied the design of a DFPSE to produce a mechanical power of 1 kW. The stability of a DFPSE is obtained taking into account various efforts like the geometric, dynamic and thermodynamic variables. Thermo-mechanical simulation makes a first study useful for the predimensioning of the free piston Stirling engine. We thus obtain from values geometrical an operating range of frequency, recovered electric power, the amplitude ratio of displacement of the pistons, the variations of the pressure in the machine. The stable operation was performed for a frequency in the range 20–35 Hz. The strokes of the piston and displacers are 2 cm and 1 cm, respectively. The mean pressure in the machine is $P_{\text{mean}} = 1.4$ MPa for a dynamic amplitude of the pressure of $\Delta P = 0.1$ MPa.

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